Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 23 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$,

(4)

(b) $\dfrac{\sin 4x}{x^3}$.

(5)

2.

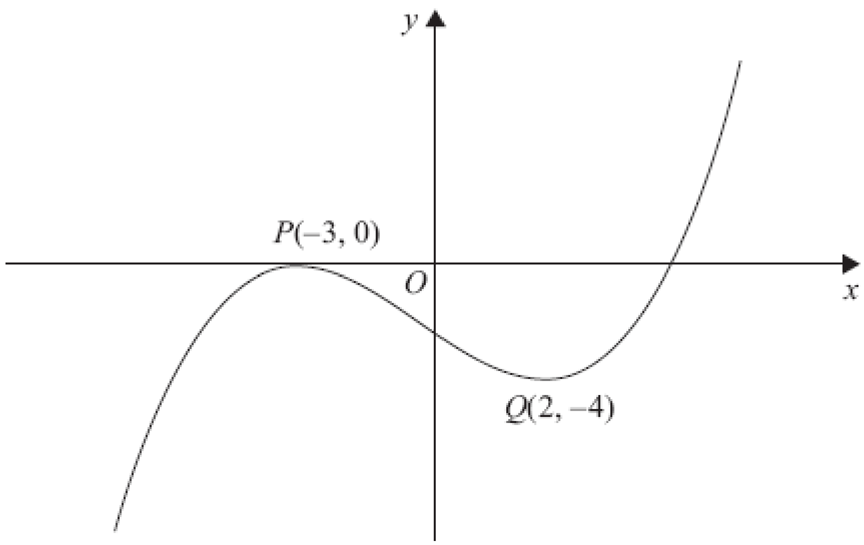

Figure 1

Figure 1 shows the graph of equation $y = \mathrm{f}(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3\mathrm{f}(x + 2)$,

(3)

(b) $y = |\mathrm{f}(x)|$.

(3)

On each diagram, show the coordinates of any stationary points.

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3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \qquad t \geq 0.$$

(*a*) Write down the area of the culture at midday.

(1)

(*b*) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

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4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

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5. Solve, for $0 \leq \theta \leq 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

(10)

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6. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \qquad 0 \le x \le \pi.$$

(*a*) Show that the equation f(x) = 0 has a solution in the interval $0.8 < x < 0.9$.

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(*b*) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}.$$

(4)

(*c*) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \qquad x_0 = 2,$$

find the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(*d*) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

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7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \qquad x \in \mathbb{R},\ x > \frac{1}{2}.$$

(*a*) Show that $f(x) = \dfrac{1}{2x-1}$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

(*c*) Find the domain of f^{-1}.

(1)

$$g(x) = \ln(x + 1).$$

(*d*) Find the solution of $fg(x) = \dfrac{1}{7}$, giving your answer in terms of e.

(4)

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8. (*a*) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

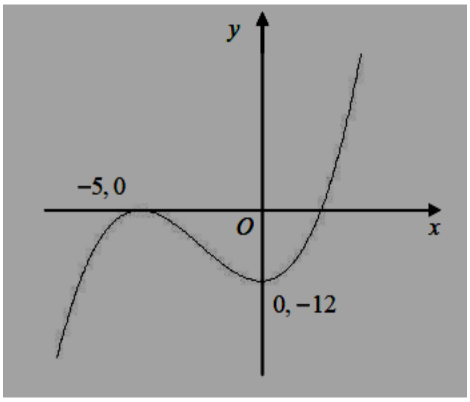
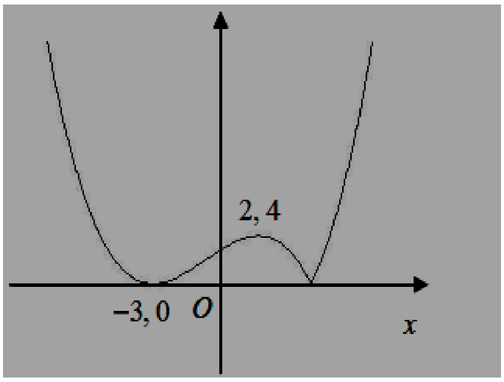
$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Give your answers as multiples of π.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\frac{d}{dx(\ln(3x))} \to \frac{B}{x}$ for any constant *B*	M1
	Applying *vu′* + *uv′*, $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
(b)	Applying $\frac{vu' - uv'}{v^2}$	
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1
		A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1 (5)
		(9 marks)
2	(a)	
		
	Shape	B1
	x coordinates correct	B1
	y coordinates correct	B1
		(3)
	(b)	
		
	Shape	B1
	Max at (2,4)	B1
	Min at (-3,0)	B1
		(3)
		6 marks

Question Number	Scheme	Marks
3. (a)	20 (mm^2)	B1 (1)
(b)	$'40' = 20\ e^{1.5t} \rightarrow e^{1.5t} = c$	M1
	$e^{1.5t} = \frac{40}{20} = (2)$	A1
	Correct order $1.5t = \ln'2' \rightarrow t = \frac{lnc}{1.5}$	M1
	$t = \frac{ln2}{1.5} = (awrt\ 0.46)$	A1
	12.28 or 28 (minutes}	A1 (5)
		(6 marks)
4.	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1, A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y = \frac{\pi}{4}$. $x = 2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m\left(x - their\ 2\sqrt{3}\right)$	M1
	$\left(y - \frac{\pi}{4}\right) = -8\left(x - 2\sqrt{3}\right)$ oe	A1
		(7 marks)
5.	**Uses the identity** $cot^2(3\theta) = cosec^2(3\theta) - 1$ in	
	$2cot^2(3\theta) = 7cosec(3\theta) - 5$	M1
	$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$	A1
	$(2cosec3\theta - 1)(cosec3\theta - 3) = 0$	M1
	$cosec3\theta = 3$	A1
	$\theta = \frac{invsin\left(\frac{1}{3}\right)}{3}$, $\frac{19.5°}{3} = awrt\ 6.5°$	M1, A1
	$\theta = \frac{180° - invsin\left(\frac{1}{3}\right)}{3}$, 53.5° Correct 2nd value	M1,A1
	$\theta = \frac{360° + invsin\left(\frac{1}{3}\right)}{3}$	M1
	Correct 3rd value	
	All 4 correct answers awrt 6.5°,53.5°,126.5° or 173.5°	**A1**
		(10 marks)

Question Number		Scheme	Marks
6.	(a)	f(0.8) = 0.082, f(0.9)= –0.089	M1
		Change of sign $\Rightarrow$ root (0.8,0.9)	A1 (2)
	(b)	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
		Sets $f'(x) = 0 \Rightarrow x = \dfrac{3 + \sin\left(\frac{1}{2}x\right)}{2}$	M1 A1* (4)
	(c)	Sub $x_0 = 2$ into $x_{n+1} = \dfrac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$	M1
		x_1 = awrt 1.921, x_2 = awrt 1.91(0) and x_3 = awrt 1.908	A1, A1 (3)
	(d)	[1.90775,1.90785]	M1
		f'(1.90775) = –0.00016... AND f'(1.90785) = 0.0000076...	M1
		Change of sign $\Rightarrow x = 1.9078$	A1 (3)
			(12 marks)

7.	(a)	$2x^2 + 7x - 4 = (2x-1)(x+4)$	B1
		$\dfrac{3(x+1)}{(2x-1)(x+4)} - \dfrac{1}{(x+4)} = \dfrac{3(x+1) - (2x-1)}{(2x-1)(x+4)}$	M1
		$= \dfrac{x+4}{(2x-1)(x+4)}$	M1
		$= \dfrac{1}{2x-1}$	A1* (4)
	(b)	$y = 1/(2x-1)$ ($y(2x-1) = 1$ ($2xy - y = 1$ $x = \dfrac{1+y}{2y}$	M1 M1
		y OR $f^{-1}(x) = \dfrac{1+x}{2x}$	A1 (3)
	(c)	$x > 0$	B1 (1)
	(d)	$\dfrac{1}{2\ln(x+1) - 1} = \dfrac{1}{7}$	M1
		$\ln(x+1) = 4$	A1
		$x = e^4 - 1$	M1 A1 (4)
			12 marks

Question number	Scheme	Marks	
8. (a)	$\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$	M1A1	
	$= \dfrac{\dfrac{sinA}{cosA} + \dfrac{sinB}{cosB}}{1 - \dfrac{sinAsinB}{cosAcosB}}$	M1	
	$(\div cosAcosB)$		
	$= \dfrac{tanA + tanB}{1 - tanAtanB}$	A1 *	(4)
(b)	$\tan\left(\theta + \dfrac{\pi}{6}\right) = \dfrac{tan\theta + \dfrac{\tan\pi}{6}}{1 - \dfrac{tan\theta\tan\pi}{6}}$	M1	
	$= \dfrac{tan\theta + \dfrac{1}{\sqrt{3}}}{1 - tan\theta\dfrac{1}{\sqrt{3}}}$	M1	
	$= \dfrac{\sqrt{3tan\theta} + 1}{\sqrt{3} - tan\theta}$	A1 *	(3)
(c)	$\tan\left(\theta + \dfrac{\pi}{6}\right) = \tan(\pi - \theta).$	M1	
	$\left(\theta + \dfrac{\pi}{6}\right) = (\pi - \theta)$	M1	
	$\theta = \dfrac{5}{12}\pi$	M1 A1	
	$\tan\left(\theta + \dfrac{\pi}{6}\right) = \tan(2\pi - \theta)$	M1	
	$\theta = \dfrac{11}{12}\pi$	A1	(6)
			(13 marks)